Digital phase shifting holography and holographic interferometry: the error analysis

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1 Introduction

Phase shifting digital holography (PSDH) technique was introduced for the first time in 1997 by I. Yamaguchi [1]. It was successfully applied to reduce unwanted zero and conjugated (twin image) terms in the reconstruction plane (Fig.1a,b) and therefore allows to reconstruct on-line holograms and use efficiently the bandwidth of a matrix detector. Also it is proven that the object phase reconstruction performed by “ideal” PSDH brings a perfect phase (the RMS and P-V errors at the level 10^{-14} (digitization errors)), while a phase from a single hologram reconstruction suffers significant P-V and RMS errors (Fig.1c).

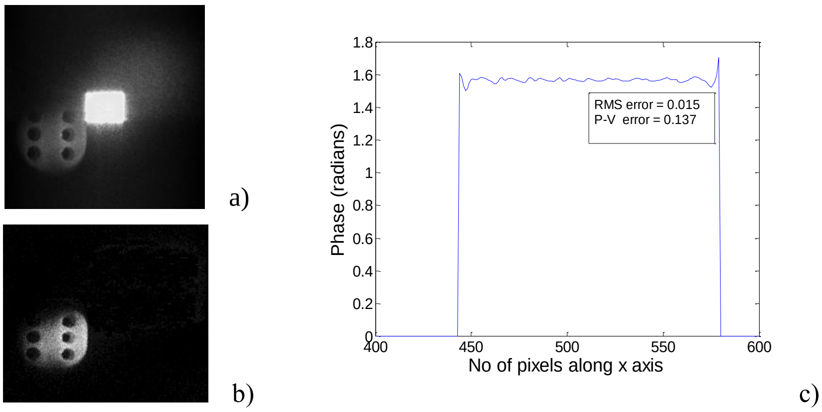


Fig. 1. The exemplary results of numerical reconstruction of off- axis Fresnel holograms a) single, b) PSDH of an intensity of a real object and c) the crossection of a constant object phase reconstructed from a single computer generated hologram

PSDH realizes a new flexibility of quantitative acquisition, processing and reconstruction of digital holograms. At the images acquisition stage PSDH is similar to temporal phase shifting interferometry TPSI [2] and requires capturing of at least three phase shifted holograms. Let us consider an object phase reconstruction (Fig.2). In the TPSI method a phase is calculated directly from a series of phase shifted interferograms (one step process), while in PSDH at first the phase of an object wavefront ϕ_H at the detector plane is calculated by the conventional phase shifting method and than an image of an object (its intensity and phase) is reconstructed by propagation of the complex amplitude U to an object plane by one of the digital holography reconstruction methods [3]. So in the case of PSDH two step reconstruction procedure is implemented. It was proven [1] that in the case of diffusely reflecting objects the phase distribution ϕ_H is enough to reconstruct images of almost the same quality.

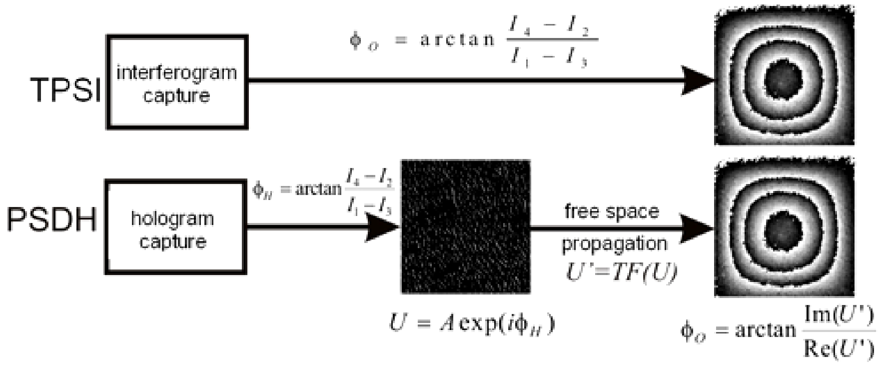


Fig. 2. The scheme of an object phase reconstruction by a) temporal phase shifting interferometry TPSI and b) phase shifting digital holography PSDH

The influence of the errors which may appear during registration of the sequential interferograms/holograms on the reconstructed object phase is well known in TPSI [2]. However in the case of PSDH they refer to the hologram phase ϕ_H but object one ϕ_O. The calculated complex wavefront U with possible errors in ϕ_H is propagated, during numerical reconstruction, between hologram and object planes and these errors will influence the errors appearing in the final reconstructed object phase. However the values and character of these errors may be different than in classical PSI. Therefore below we present the extended investigation of the influence of typical PSI errors on the phase determined by PSDH.

2 Comparison of the errors in PSI and PSDH

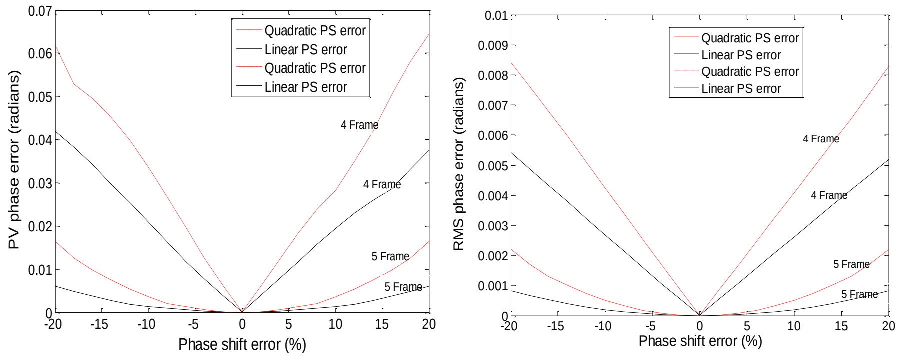
The most important sources of errors in the phase shifting interferometry include: miscalibration of a phase shifter, nonlinearities due to detector and air turbulences and vibrations. Those and many other errors were studied by several researchers and are well described [2,4]. In order to compare the influence of these errors on the accuracy of phase reconstruction by PSI and PSDH we choose a flat reflective object with $\phi_O = \pi/2$. The considered linear ε_p and quadratic ε_n phase shift errors and the nonlinearity of detector error are in the range +/-20% [2]. Additionally we will compare the performance of 3-, 4- and 5-frame phase shifting algorithms. The object phase retrieval in the second step of PSDH is performed by the angular plane waves spectral decomposition method [5].
At first the influence of the linear phase-shift error ε_p (Fig.3a)

$$\varphi' = \varphi(1+\varepsilon_p) \tag{1}$$

and the non-linear phase shift error ε_n (Fig.3b)

$$\varphi' = \varphi(1+\varepsilon_n\varphi) \tag{2}$$

have been investigated.



a) b)

Fig. 3. Object phase errors due to linear and nonlinear phase shift error obtained in 4-frame and 5-frame PS algorithms: a) P-V values and b) RMS values

In both cases we can conclude that the object phase errors (both RMS and P-V) are much smaller (more than one order of magnitude) than in the case of PSI, however they maintain similar character. Also the values of errors obtained for significant phase shift error (20%) are much smaller than in the case of the object phase reconstruction from a single hologram (compare with Fig.1c). In the case of second order nonlinear detector

errors the object phase reconstructed by 4- and 5-frame algorithms have, like in the case of PSI, negligible errors, while three frame algorithm generate three times smaller errors than in PSI [6].
Finally we can conclude that in the case of a reflective (transmissive) object (e.g. the case of DH microscopy) PSDH is less sensitive to typical errors which may be introduced during capturing of phase shifting holograms. As in the case of PSI the five frame algorithm is most favorable to obtain high accuracy reconstruction of an object phase. However even the four frame and three algorithms give considerable improvement over the accuracy of phase reconstructed from a single hologram.

3 PSDH for arbitrary object phase reconstruction and phase shifting digital holographic interferometry

The analysis performed in Section 2 presents the results for a very simple object. In practice objects have complex phases and, most often, scattering properties (diffuse objects). For these reasons we investigate the errors in PSDH for the following cases:

- the reflective objects with linear phases introduced in the range from $\pi/2$ to 7π rad (Fig.4),
- the scattering object (random phase added) with linear phase 0- $\pi/2$ rad (Fig.5).

The numerical simulations were performed for 4-frame algorithm, which is commonly used in DH. The 5-frame algorithm errors will be smaller, so we can use it if the highly accurate measurements are required.

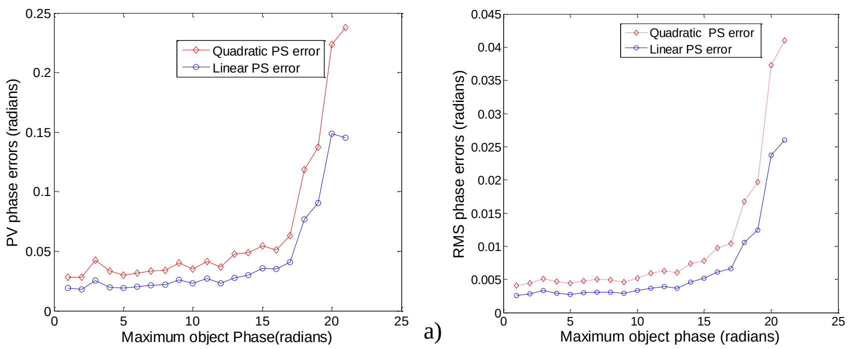


Fig. 4. The reflected object phase error in the function of the maximum value of object linear phase: a) P-V phase error and b) RMS phase error calculated for the case of 10% linear and quadratic phase shift error

It is clearly seen from Fig.4 that the object phase error is nearly constant up to a certain phase gradient value (here up to 15rad), for bigger phase gradients the error increases significantly. This fact has to be taken into account in the case of an measurement uncertainty determination of an object with complex phase.
The second case considers the PSDH applied for determination of the phase of a scattering object. The numerical analysis (with known model of theoretical object phase values with the random phase) allows to determine the reconstructed object phase errors for different values of linear and quadratic phase shift errors (Fig.5).

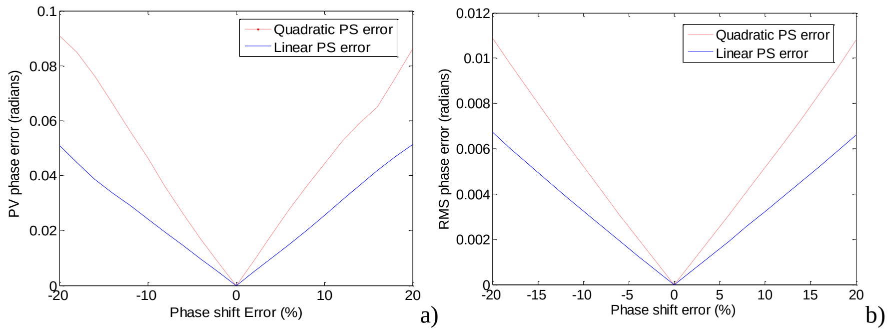


Fig. 5. The scattering object phase error in the function of linear and quadratic phase shift error: a) P-V phase error and b) RMS phase error calculated for the case of linear object phase ($\phi_{Omax}=\pi/2$).

The reconstructed object phase errors have in this case bigger values, when compared with those obtained for a reflective object, however still these values are smaller than in PSI. Of course in a real experiment the phase obtained from a scattering object cannot be used directly as it contains a random phase. In the metrology of scattering object we use holographic interferometry. In the digital holographic interferometry, the initial phase of an object is subtracted from the sequential phases of the object under load [3]. The result of such subtraction between two linear object phases ($\phi_{O1max}=\pi/2$, $\phi_{O2max}=3\pi$) are shown in Fig.6 while the crossections of the phase error maps are presented in Fig.7. The single phases were calculated by 4-frame PS algorithm and for the 10% linear and quadratic phase shift errors.

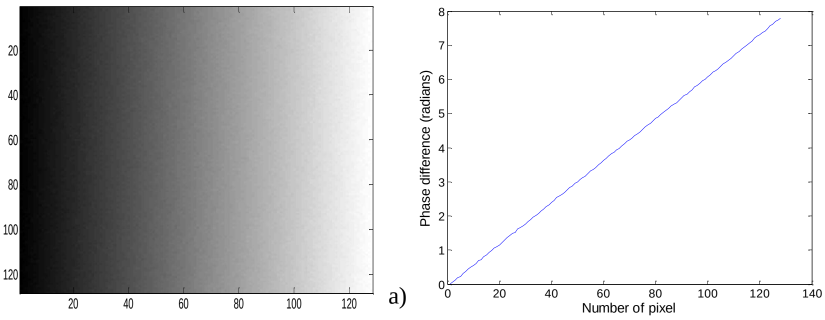


Fig. 6. The result of the phase shifting digital holographic interferometric procedure: a) the object phase difference and b) its horizontal crossection.

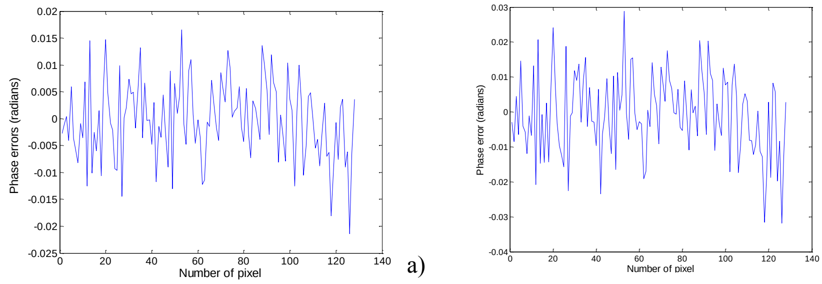


Fig. 7. The crossections of the object phase difference errors: a) for 10% linear PS error and b) 10% quadratic PS error

The theoretical RMS and P-V errors of the reconstructed phase difference are at the very similar level as those obtained for a single phase reconstruction by PSDH (compare with Fig.5). However in real holographic interferometry experiments we can expect higher level of errors especially due to the random phase term caused by an environmental instability.

4 Conclusions

The phase shifting digital holography PSDH provides an excellent improvement of both digital holography and digital holographic interferometry. It allows to obtain a high quality object phase reconstruction without influence of the zero and conjugate terms. It was shown that, due to the two step process in an object phase reconstruction, PSDH has much smaller sensitivity to the typical PSI errors. It refers to both reflective/transmissive and scattering objects, therefore it can be successfully applied for high accuracy quantitative phase analysis in digital holographic microscopy and digital holographic interferometry.

5 Acknowledgments

The research leading to these results has received funding from the European Community's Seventh Framework Programme FP7/2007-2013 under grant agreement n° 216105 (“Real 3D” Project).

6 References

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