RECONSTRUCTION OF FRESNEL DIGITAL HOLOGRAM USING FRACTIONAL FOURIER TRANSFORM ALGORITHM

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Abstract: We report the reconstruction of a digital hologram, recorded in the Fresnel domain, using the fractional Fourier transform algorithm. In the algorithm, only fractional order is needed for satisfactory reconstruction instead of using the parameters such as distance, wavelength, and resolution of recording sensor required in the Fresnel reconstruction.

1. INTRODUCTION

The research of fractional Fourier transform (FRT) in recent years has made great improvement bringing new opportunities for the digital and optical information processing technologies. The FRT was used initially as a mathematical tool in quantum mechanics [1] and later in optics [2-4]. It is defined as a generalization of the ordinary Fourier transform with an order parameter *p*. The FRT is physically generated by diffraction and electromagnetic wave propagation. Researchers [5-10] have investigated the relation among the FRT, the Fresnel transform, and the Fraunhofer transform. The continuity of FRT corresponds to the continuity of wave propagation from no diffraction to the Fraunhofer diffraction passing by the Fresnel diffraction. FRTs are related to Fresnel diffraction in the same way as the standard Fourier transform is related to Fraunhofer diffraction. Since both FRT and Fresnel transform describe light propagation, it is clear that they must be related. The relationship between the FRT and the free-space propagation integral has been stated in many contributions through different approaches. It has been demonstrated that the Fresnel diffraction can be expressed as an FRT, whose orders depend on the distance between the object and the observation field [5,6].

Lohmann [3] proposed that the FRT could be optically realized by using the conventional lens system. He proposed two optical setups for performing the FRT. James and Agarwal [7] considered the Fresnel transform and the FRT as the special cases of the generalized Fresnel transform. It was later demonstrated that the FRT can be performed by free-space Fresnel diffraction that is by without the use of a lens system [8]. It has also been proposed that the FRT can be performed by using Fresnel diffraction and one lens system [9]. Four different schemes have been suggested for the optical implementation. With this optical realization of FRT of an object, the scaling comes into the picture. It has been demonstrated that the Fresnel diffraction pattern caused by an object is an enlarged pattern of the FRT of the same object [10]. Physically, it relates the optical diffraction between two asymmetrically positioned planes before and after a lens. Further, the concept of an extended or generalized FRT has been suggested with more parameters and wider domain [11]. A flexible design employing three generalized lenses and two free-space intervals have also been proposed [12]. This design permits one to change the transformation angles by the proper lens rotation without depending on scaling and additional phase modulation.

Several advantages can be obtained from the Fresnel-FRT relation. FRT provides a compact and coherent way of describing the scalar diffraction problem and introduces a new perspective into signal processing in domains different than object or Fourier domains [13]. Also it can be accurately calculated through fast numerical algorithms [14,15]. On the other hand, the Fresnel integral provides a more intuitive description of an optical signal. Recently, FRT has been combined with digital holography and used in several applications, such as, analyzing the motion of a surface [16], anti-counterfeiting [17], analysis of the diffraction patterns of a particle field hologram [18] generated by elliptic and astigmatic Gaussian beams [19], optical information security [20], and in reconstruction of particle holograms recorded with a pulse laser beam [21]. Combining the benefits of both the Fresnel and the FRT, in this paper, we present reconstruction of a phase-shift digital hologram recorded in the Fresnel geometry, using FRT algorithm. We propose that the preliminary results of this experiment would find application in holographic 3D display [22,23]. The proposed optical architecture would always reconstruct the Fresnel hologram at a constant reconstruction distance when it may have been captured for different objects at different distances.

2. FRACTIONAL FOURIER TRANSFORM

A two-dimensional FRT of a function *f(x,y)* of order ($\alpha = a\pi/2$) is given by $g(\xi,\eta)$ as [4]

$$g(\xi,\eta) = K\iint f(x,y) \times \exp\left(j\pi \frac{x^2+y^2+\xi^2+\eta^2}{\tan\alpha} - 2j\pi \frac{xy\xi\eta}{\sin\alpha} \right) dxdy \quad (1)$$

Here (x,y) and (ξ,η) represent the space and fractional domain coordinates, respectively. The parameter K is a complex constant.

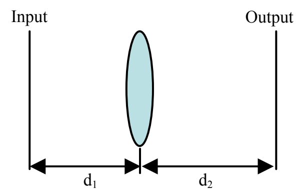

Fig. 1 Scheme to optically implement the FRT.

For optical implementation of FRT, Lohman proposed two optical setups, called type-I and type-II setups [3]. But in these setups free-space propagation distance, both sides of the lens system are same. For digital hologram reconstruction, we wish to use a system where the viewer's position always remains fixed but the fractional order should be changed. We propose to use the optical setup shown in Fig. 1 as proposed by [11]. With the use of a liquid crystal lens, whose focal length can be varied, it may be possible to optically reconstruct the digital hologram at the viewer's desired position. The extended FRT is defined as [11],

$$u(y) = u[v(x)|a,\phi,b|y] = K\int_{-\infty}^{\infty} v(x) \times \left(i\pi \frac{a^2x^2+b^2y^2}{\tan\phi} - i2\pi \frac{ab}{\sin\phi} xy \right) dxdy \quad (2)$$

where a, b, and □ are the three quadratic phase system parameters of the extended FRT and they are related to the distances d_1 and d_2 and the focal length f of the lens through the expressions.

$$a^2 = \frac{1}{\lambda} \frac{\sqrt{f-d_2}}{\sqrt{f-d_1}} \frac{1}{\left[f^2-(f-d_1)(f-d_2)\right]^{1/2}} \quad (3)$$

$$\phi = \arccos\left(\frac{(f-d_1)(f-d_2)}{f} \right) \quad (4)$$

$$b^2 = \frac{1}{\lambda} \frac{\sqrt{f-d_1}}{\sqrt{f-d_2}} \frac{1}{\left[f^2-(f-d_1)(f-d_2)\right]^{1/2}} \quad (5)$$

Performing an extended FRT on a function is equivalent to expanding the function a times, performing an FRT of order φ, and contracting the resultant distribution b times. In our computation, the object function v(x) is replaced with the digital hologram function, say *I(x)*.

$$u(y) = K\int_{-\infty}^{\infty} I(x) \times \left(i\pi \frac{a^2x^2+b^2y^2}{\tan\phi} - i2\pi \frac{ab}{\sin\phi} xy \right) dx \quad (6)$$

We compute the FRT of *I(x)* at different fractional orders that gives us the reconstructed images of the 3D object or 3D scene. At some particular fractional order a specific object/scene from the 3D scene could be well focused/localized. Researchers have investigated the relationship between Fresnel diffraction and FRT [5-11] and their ways of definition and implementation varies, but they all produce similar results. It has been shown that the Fresnel diffraction intensity pattern of an input object $u_0(x)$ on the output plane x_1 at a distance z from the object is mathematically equal to an FRT of order p, where $\varphi = p\pi/2$ [8]. To get this mathematical relationship, what is needed is to scale the input object $u_0(x)$ by a factor cos φ times its original size and then place the reduced object $u_0(x/\cos\varphi)$ on the input plane of the optical system. The Fresnel diffraction distance is $z = f_e \sin\varphi \cos\varphi$. f_e is called the standard focal length. In other words, the Fresnel diffraction pattern caused by an object is an enlarged pattern of the FRT of the same object.

With our experimental data, we established a numerical relationship between the Fresnel distance and the orders of FRT. Consider one of the holograms, whose reconstruction has been shown in Fig. 2(a), say chip4. The hologram was recorded with the phase-shift Fresnel geometry at the recording distance, z = 178 mm. The hologram was reconstructed successfully using FRT algorithm with fractional order, p = 0.30. Here, $\varphi = 0.15\pi$ and $z = f_e \sin\varphi \cos\varphi$, therefore, the standard focal length, $f_e = z/\sin\varphi\cos\varphi = 440$ mm, and the focal length of the lens, $f = f_e/\sin\varphi = 969$ mm. Thus, a digital hologram recorded at a distance of 178 mm using Fresnel transform will optically be reconstructed at a fractional order of 0.30 and with a lens of focal length 969 mm. The values mentioned above might change with the use of different algorithm or with different way of implementation. We followed the principle and notations proposed in Ref. 8.

3. EXPERIMENT

We used real-world 3D scenes captured by phase-shift Fresnel digital holography. Our procedure to capture the 3D scene is the same as that described previously [24]. The laser source was radiating a beam at 632.8 nm. The recording camera had a resolution of 2048 × 2032 pixels with a pixel pitch of 9 µm. The numerical simulation has been carried out using Matlab. We present digital reconstructions of four different Fresnel digital holograms using FRT algorithm. To determine the position of the object, the fractional order was scanned [25]. Figs 2(a-h) show the intensity reconstructions at different fractional orders from the 2048 × 2048 pixel optically captured different holograms. Figs. 2(a) and (b) show the reconstructed intensity images of a 3D object called chip4, with fractional orders 0.30 and 0.31, respectively.

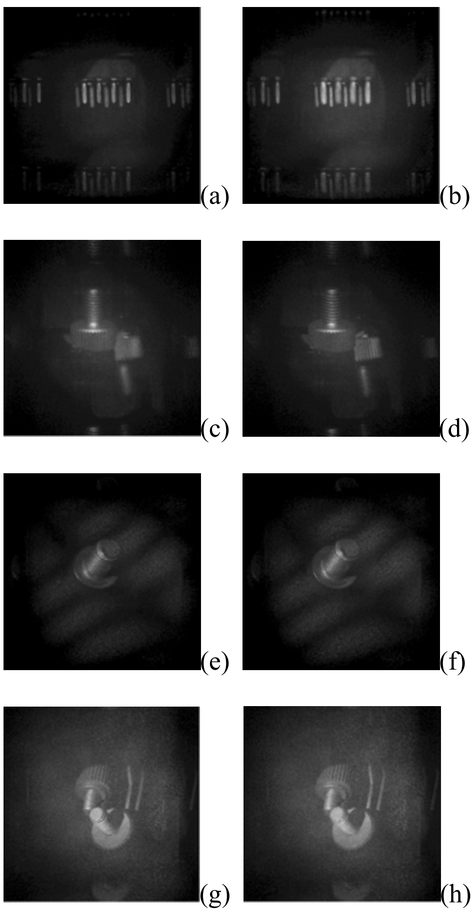

Fig. 2 Simulation results. Intensities of images reconstructed from Fresnel digital holograms using FRT algorithm: (a) chip4, z = 178 mm, p = 0.30, (b) chip4, z = 186 mm, p = 0.31, (c) TwoScrewsDepthFara, z = 350 mm, p = 0.495, (d) TwoScrewsDepthFara, z = 366 mm, p = 0.51, (e) LargeScrew, z = 330 mm, p = 0.475, (f) LargeScrew, z = 337 mm, p = 0.485, (g) TwoScrews3a, z = 357 mm, p = 0.50, and (h) TwoScrews3a, z = 365 mm, p = 0.51.

The holograms were recorded at Fresnel distances, 178 mm and 186 mm, respectively. Figs. 2(c) and (d) show the reconstructed intensity images of a 3D object called TwoScrewsDepthFara, with fractional orders, 0.495 and 0.51, respectively. The holograms were recorded at a Fresnel distances, 350 mm and 366 mm, respectively. Figs. 2(e) and (f) show the reconstructed intensity images of a 3D object called LargeScrew, with fractional orders, 0.475 and 0.485, respectively. The holograms were recorded at a Fresnel distances, 330 mm and 337 mm, respectively. Figs. 2(g) and (h) show the reconstructed intensity images of a 3D object called TwoScrews, with fractional orders, 0.50 and 0.51, respectively. The holograms were recorded at a Fresnel distances, 357 mm and 365 mm, respectively. Reconstruction with different fractional orders helps focus the different perspectives of the 3D image. Fig. 3 shows a plot between the experimental Fresnel distance and the fractional order parameter used in simulation for reconstructing the 3D object. It can be observed that there is almost a linear relationship between them.

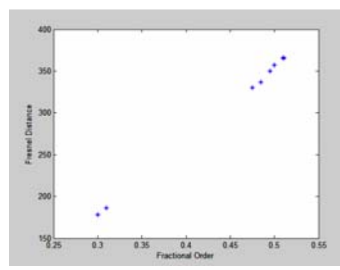

Fig. 3Plot: Fresnel distance (experimental data) versus fractional order (simulated data).

4. CONCLUSION

We present the results of reconstruction of phase-shift Fresnel digital holograms using the FRT algorithm. The algorithm requires only the fractional order information for satisfactory reconstruction as compared to the Fresnel reconstruction algorithm, where Fresnel parameters such as recording distance, wavelength of the source, resolution and bandwidth of recording sensor, are required. Implicitly, each of these parameters is set to unity in our FRT algorithm. As a proof-of-concept the simulation results are presented. The FRT can also be used to search the location of a particular object in a 3D scene [25]. We hope that the proposed technology will find application in holographic 3D displays.

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