Compensation of reference beam sphericity in a multi-perspective digital holography based record-display setup.

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Abstract

The presence of a residual sphericity in a reference beam causes magnification in the reconstructed image in digital holography. We discuss a method to estimate the relative sphericities in a multi-perspective multi-camera digital hologram recording unit. The digital holograms can then be compensated with numerical quadratic phase factors so that the object appears with the same magnification in all the reconstructions.

Keywords: Digital holography, 3D display, sphericity compensation.

1. INTRODUCTION

Holography is a technique to capture and replay amplitude and phase information of 3D objects. Digital holography[1-4] has many applications in metrology[5], shape measurement[6], microscopy[7] and in 3D display[8]. In digital holography of 3D objects, a coherent reference wave is incident on a 2D image sensor array where it forms an interference pattern with coherent light reflected of the object. The pixel pitch, the pixel size and the total number of pixels in the imaging array play a role in limiting the resolution and the size of the object which can be recorded at a particular distance[9]. The resulting pattern, called the digital hologram (DH), contains encoded information about the 3D topographic features of the object. The object of whose the hologram has been recorded can be reconstructed numerically or optically. The numerical reconstruction is typically done by simulating a Fresnel transform. Figure 1 shows a schematic of the recording and numerical reconstructions

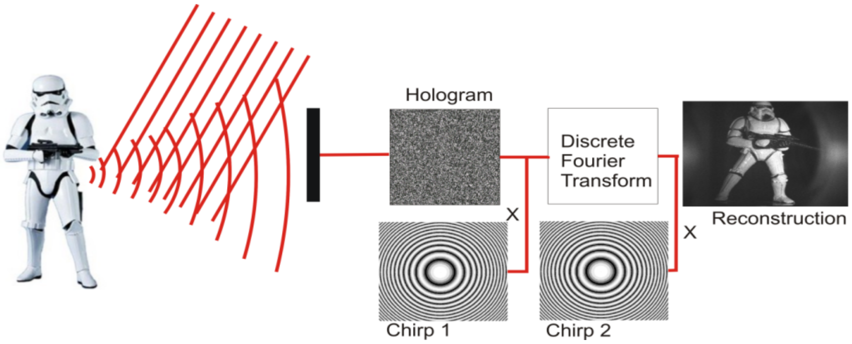

Figure 1.Recording and numerical reconstruction of digital holograms.

The object can also be replayed optically by displaying the hologram on a Spatial light modulator (SLM). Such a display system comprises of a recording unit and a display setup. The recording unit records digital holograms of a 3D object, processes them and transmits them to a display setup. The digital hologram recording unit comprises of an interferometer or a set of interferometers. A set of hologram recorders placed around the object enables us to capture holograms from a wider angular perspective. The display side comprises of spatial light modulator or an arrangement of them.

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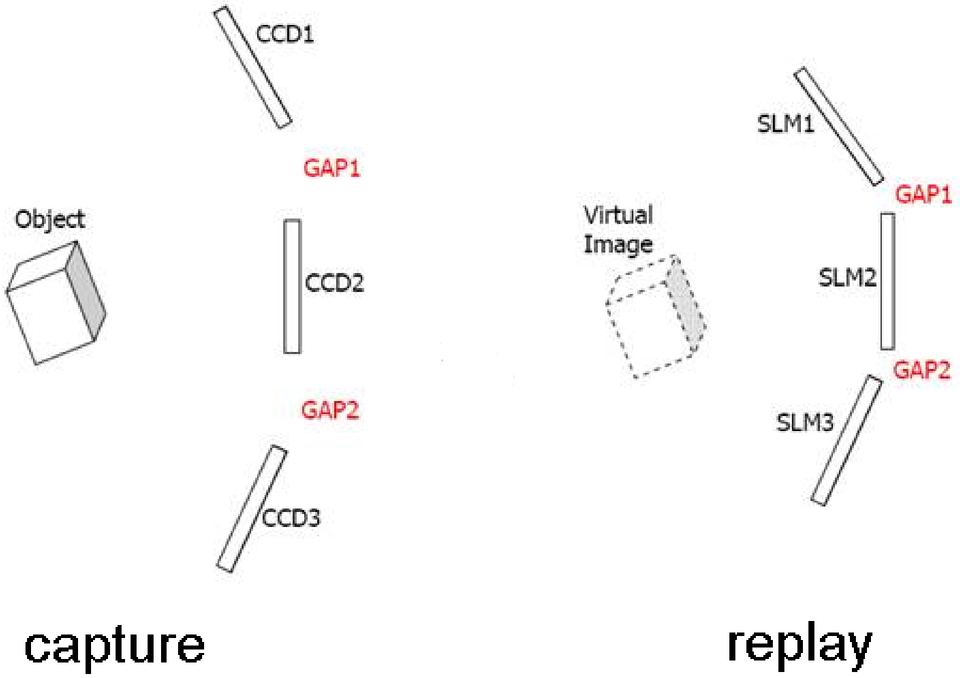

Figure 2. Recording and display architecture of digital holograms.

The holograms are displayed on the SLM arrangement and when observed through the SLM, a 3D object is reconstructed. The FP7 project, 'REAL3D' is concerned with this application of digital holography[10]. The recording side is made up of an array of hologram recorders which are placed around an object. Each recorder generates its own reference beam and records a hologram of the object from a different perspective. These holograms are then displayed on a SLM with a matching arrangement. Figure 2 shows a schematic of such an arrangement. In any practical holographic recording setup, the reference beam can have a residual sphericity in spite of good alignment of the collimating lenses. The different sphericity in each of the recorders causes the reconstruction parameters to change. It causes a change in the magnification and in the focusing distance. If these sphericities are compensated, then the object will appear at the same distance and with the same magnification and the quality of 3D reconstruction in a multi-SLM display can be improved. We have developed a procedure with which we can estimate the relative sphericities in the reference beams and then compensate each hologram with a numerical quadratic phase factor. In section 2, we discuss the theory and the methodology involved. And in section 3, we discuss the results obtained from a multi-capture arrangement at BIAS and conclude.

2. METHODOLOGY

To accurately measure the sphericity of the reference beam requires interferometric techniques. It is however possible to estimate the relative sphericities in the reference beam by using simple geometrical techniques. Our technique is based on the ideas of recording digital holograms of a known object with the recording arrangement and then estimating the magnification in the reconstruction. We attribute the magnification mismatch to a residual sphericity in the reference beams and estimate it.

Assuming that we have an ideal on axis plane wave reference beam to record the digital hologram, the Fresnel transform relates the recorded hologram to an object plane at a distance *z* away

$$object(x',y') = recon(x',y') = F_z\{C(x,y)\}(x',y') \int_{-W/2}^{W/2} C(x,y)\exp\left(\frac{j\pi}{\lambda z}\left[x^2 - 2xx' + x'^2 + y^2 - 2yy' + y'^2\right]\right) dxdy$$

where *object(x',y')* represents the complex object wave, *recon(x',y')* represent the ideal reconstruction, *C(x',y')* represents the hologram obtained at the camera plane and F_z is the operator representing the Fresnel transform. For ideal in line plane wave reference beam, the image of the object will be found in focus at the reconstruction distance equal to the object–camera distance. However, if there exists an off axis reference beam with a (diverging) spherical curvature, which is not centred we can describe the reference as follows

$$ref(x,y) = \exp\left(\frac{j\pi\alpha}{\lambda}\left[[x-\mu_x]^2 - \left[y-\mu_y\right]^2\right]\right)\exp\left(\frac{j2\pi}{\lambda}\left[x\xi_x + y\xi_y\right]\right)$$

Where (μ_x, μ_y) is the centre of curvature, α represents the curvature, and ξ_x and ξ_y represent the linear phase term. This reference beam can be rewritten as follows

$$ref(x,y) = \exp\left(\frac{j\pi\alpha}{\lambda}[x^2 + y^2]\right)\exp\left(\frac{j2\pi}{\lambda}\left[x\kappa_x + y\kappa_y\right]\right)$$

We have omitted a constant phase term and

$$\kappa_x = \xi_x - \alpha\mu_x$$

$$\kappa_y = \xi_y - \alpha\mu_y \qquad (1)$$

Illumination with such a reference beam will provide the following reconstruction at an arbitrary distance *z'*

$$F_{z'}\{C(x,y)ref(x,y)\}(x',y') =$$

$$\int_{-W/2}^{W/2} C(x,y)ref(x,y)\exp\left(\frac{j\pi}{\lambda z'}\left[x^2 - 2xx' + x'^2 + y^2 - 2yy' + y'^2\right]\right) dxdy$$

$$= \int_{-W/2}^{W/2} C(x,y)\exp\left(\frac{j\pi}{\lambda z'}[x^2 + y^2][1 + z'\alpha]\right)\exp\left(-\frac{j2\pi}{\lambda z'}\left[x[x' - z'\kappa_x] - y\left[y' - z'\kappa_y\right]\right]\right)\exp\left(\frac{j\pi}{\lambda z'}\left[x'^2 + y'^2\right]\right) dxdy$$

Let

$$z' = \frac{z}{1 - z\alpha} \qquad (2)$$

In this case our reconstruction at distance z' can be rewritten as follows

$$= \int_{-W/2}^{W/2} C(x,y) \exp\left(\frac{j\pi}{\lambda z}[x^2+y^2]\right) \exp\left(-\frac{j2\pi}{\lambda z}\left[x\left[\frac{x'}{M} - z\kappa_x\right] - y\left[\frac{y'}{M} - z\kappa_y\right]\right]\right) \exp\left(\frac{j\pi M}{\lambda z}\left[\left(\frac{x'}{M}\right)^2 + \left(\frac{y'}{M}\right)^2\right]\right) dxdy$$

where

$$M = \frac{z'}{z} = \frac{1}{1 - z\alpha} \tag{3}$$

If we interest ourselves only with the intensity of this reconstruction we can ignore the last phase term. We can write that the intensity of the term above is given as follows

$$\left|F_{z'}\{C(x,y)ref(x,y)\}(x',y')\right|^2 = \left|F_z\{C(x,y)\}\left(\frac{x'}{M} - z\kappa_x, \frac{y'}{M} - z\kappa_y\right)\right|^2$$

This means that

- The spherical curvature will cause our reconstruction to appear in focus at a distance *z'* when the object was positioned at *z*. The relationship between *z* and *z'* is given by Equation (2) above.
- Furthermore the reconstruction in this plane will be magnified by an amount *M* which is dependent on the spherical curvature and also on z. It is given in Equation (3) above.
- The reconstruction is shifted in *x',y'* by an amount $z\kappa_x$ and $z\kappa_y$ which we recall is dependent on the linear phase and on the curvatures centre position on the camera as shown in Equation (1)

In the method described above, we use an object whose size is known to us. The object is also chosen to be planar to have fewer speckles and has features which can be easily identified. Figure 1 shows the object which we have chosen.

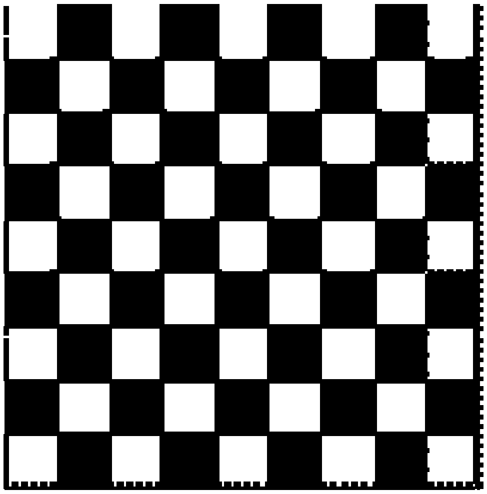

Figure. 3. The checkerboard object used in the experiments. (18.9cm x 18.9 cm)

3. EXPERIMENTS AND RESULTS

An experimental setup for recording digital holograms of objects from multiple perspectives has been developed at BIAS[11]. The object shown in Figure 3 was printed at 600dpi and was recorded with the six cameras. Ten statistically different holograms were captured by diffusely illuminating the object. The object is reconstructed and then the independent reconstructions are added on an intensity basis to improve the reconstructed image quality.

The following steps are taken to find out the sphericities of each reference beam.

1. The focusing distance, d_{focus} is identified by using a variance based auto-focus method. Three different regions in the object are chosen and the average is taken of the respective focusing distances. Figure 3 shows the results of the autofocussing method on hologram obtained from camera 1.

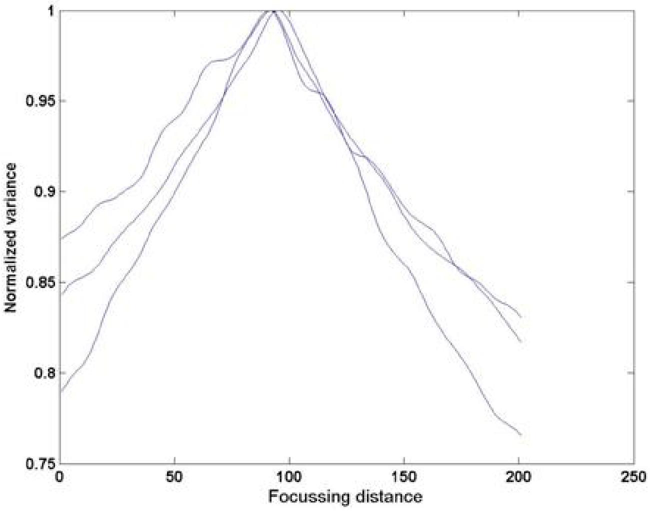

Figure 3. Detected focus points of 3 regions in the reconstruction.

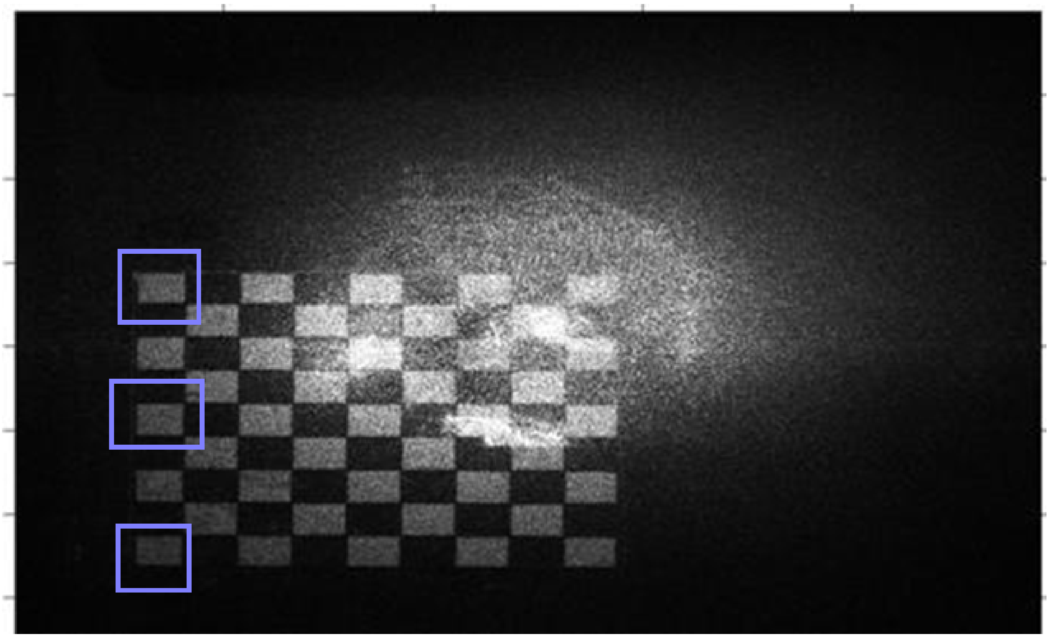

Figure 4. Reconstructed image. The 3 regions selected for autofocussing are shown.

2. The corresponding pixel pitch is found by using the pixel pitch relationship in the Fresnel plane.

$$P_x = \frac{\lambda d_{focus}}{N\Delta x} \tag{4}$$

3. The size of the chart as detected the observer is found by counting the number of pixels used up in imaging the side of the chart. The side most edge is taken in this case. Thus the length of the chart detected by the observer is found out.

4. The actual real world length of the edge is known since the object is known.This is a 9x9 chess board pattern. Each square is 0.21 cm. The length of each edge is 1.89cm

5. Thus the supposed pixel pitch which would give the true length in the reconstruction is found out and the focusing distance $d_{supposed}$ corresponding to this pitch is found.

6. The mismatch is assumed to be because of the reference beam and the sphericity ($1/d_r$) is calculated.

$$\frac{1}{d_r} + \frac{1}{d_{focus}} = \frac{1}{d_{supposed}} \tag{5}$$

This process is repeated for all the 6 cameras. The results are tabulated below.

Table.1

Camera	Focussing distance(mm)	Pixel pitch(um)	Actual length(mm)	Detected length(in pixels)	Detected length in mm	Correct pixel pitch	Correct focusing distance(mm)	Corrective Sphericity $(\times 10^4$ mm)
1	179	27	189	696	187	26.8	180	-3.22
2	177.8	26.8	189	695.21	186.31	27.18	180	-1.45
3	178.2	26.9	189	691.12	185.9	27.34	181.05	-1.13
4	179.7	27.13	189	693.01	188.01	27.27	180.59	-3.64
5	178.1	26.89	189	694.05	186.63	27.23	180.33	-1.44
6	179.56	27.11	189	694.28	188.21	27.22	180.13	-5.67

4. CONCLUSION

One of the issues when recording digital holograms of the same object with multiple recorders is the presence of different sphericities in the reference beam in each interferometer. This causes the reconstruction of the same object to appear different from hologram. We have discussed the impact of sphericity mismatch in the reference beams on the reconstructed image magnification. It is of interest to estimate these mismatches and adequately compensate for them before display and we have presented a method to estimate these values. Our method relies on detecting the magnification and focusing depth of the images of a known object and then estimating the reference beam curvature.

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